Principles of Energy Balance in Environmental Systems

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Lecture 6

1. **Conduction and convection**
2. Calculating boundary layer conductance
3. Measurement of the driving gradient
4. Modelling convective heat transfer

HEAT TRANSFER BY CONDUCTION AND CONVECTION CAN BE DETERMINED BY:

$$HT = 2 * K * \frac{\Delta T}{BL}$$

Where:

HT = Heat transfer in watts m^{-2} (ALSO CALLED C^2)

2 = Each leaf has two sides and both sides can transfer heat

K′ = The thermal conductivity coefficient of air, which is 0.026 W m^{-1} $°c^{-1}$. This varies slightly with temperature and moisture content of the air, but we will consider it a constant.

ΔT = Delta temperature - difference between leaf temperature and air temperature This can be either positive or negative (unit = °c or k).

BL = Air boundary layer. This is the thickness of the stagnant air layer next to the leaf (unit = meters). Note that typical boundary layers are 0.0001 to 0.01 meters. BL thickness is determined by:

$$\underset{\text{(meters)}}{BL} = 0.004 \sqrt{\frac{leaf\ width\ /\ meters}{air\ velocity\ /\ meters\ per\ second}}$$

Typical values for air velocity (wind speed) are:

0.1 m s^{-1} = still air
10.0 m s^{-1} = high wind (22 mph)

TABLE SHOWN ON PAGE 2

Thickness in mm of the boundary layer adjacent to a leaf, δ^{bl}_{mm}, for a variety of leaf sizes and wind speeds. Calculations were made assuming that $\delta^{bl}_{mm} = 4.0 \cdot \text{sqrt}(l_{(m)}/v_{(ms^{-1})})$. (Eq 7.7, where $l_{(m)}$ is the mean leaf length in the wind direction in m and $v_{(m\,s^{-1})}$ is the ambient wind speed in m s^{-1}. (Note that 1 km $hour^{-1}$ = 0.278 m s^{-1}, and 1 mile $hour^{-1}$ = 0.447 m s^{-1})

		WIND VELOCITY $v_{(ms^{-1})}$ (meters per s)						
		0.10	0.28	0.45	1.00	2.78	4.47	10.00
LEAF WIDTH OR $l_{(m)}$ LENGTH	0.002	0.57	0.34	0.27	0.179	0.107	0.085	0.057
	0.01	1.26	0.76	0.60	0.40	0.24	0.189	0.126
	0.05	2.8	1.69	1.33	0.89	0.54	0.42	0.28
	0.25	6.3	3.8	3.0	2.0	1.20	0.95	0.63
	0.50	8.9	5.3	4.2	2.8	1.70	1.34	0.89

An example calculation:

Measured parameters:

Air temp = 20°
Leaf temp = 22° } $\Delta T = 2^\circ C$
Leaf width = 0.05 m (5 cm)

Air velocity = 1 m s^{-1}

$$BL = 0.004 \sqrt{\frac{0.05}{1}}$$

$$= 0.000894 \text{ m}$$

$$= 0.89 \text{ mm}$$

$$HT = 2 \cdot 0.026 \cdot \frac{2}{0.000894}$$

$$= 0.052 \cdot 2236$$

$$\boxed{= 116 \text{ W m}^{-2}}$$

Heat transfer is determined by: $\dfrac{\Delta T \cdot \sqrt{air\ velocity}}{\sqrt{leaf\ width}}$

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A SIX STEP PROCEDURE TO DETERMINE THE ENERGY BALANCE COMPONENTS OF A LEAF

$\text{Ø}_{\text{IN}} = \text{Ø}_{\text{OUT}}$ + Transpiration + Condition/Convection

STEP 1. Determine Incident Radiation

$\text{Ø}_{\text{incident}} = \text{Ø}_{\text{shortwave}} + \text{Ø}_{\text{short/reflected}} + \text{Ø}_{\text{longwave/sky}} + \text{Ø}_{\text{long/ground}}$

Ø_{s} = Measure directly

$\text{Ø}_{\text{s/ref}}$ = Calculate (Ø_{s} * Albedo of ground)

$\text{Ø}_{\text{L/sky}}$ = MEASURE ~~Calculate (Ø_{total} - Ø_{s} -~~ ($\text{Ø}_{\text{L/sky}}$)

$\text{Ø}_{\text{L/ground}}$ = Calculate ($\text{Ø}_{\text{emmitted}} = e \cdot \sigma \cdot T_k^4$)
Based on ground temperature

STEP 2. Determine Absorbed Radiation

$\text{Ø}_{\text{absorbed}} = \text{Ø}_{\text{incident shortwave}}$ * (% $\text{absorbed}_{\text{shortwave}}$)

+ $\text{Ø}_{\text{incident longwave}}$ * (longwave absorptivity)

STEP 3. Determine $\text{Ø}_{\text{emmitted}}$ (Ø_{out})

$\text{Ø}_{\text{emmitted}} = e \cdot \sigma \cdot T_k^4 \cdot 2$ Leaves have two sides and can emit radiation from the top and bottom

STEP 4. Determine Ø_{net}

$\text{Ø}_{\text{net}} = \text{Ø}_{\text{absorbed}} - \text{Ø}_{\text{emmitted}}$

STEP 5. Determine Energy Transfer by Latent Heat of Evaporation (Transpiration)

$$Flux = \frac{Driving\ Gradient\ (DG)}{Resistance} = DG * Conductance$$

$$Transpiration\ Rate = \frac{SWVP_{leaf} - WVP_{air}}{Resistance}$$

Calculate energy removed by transpiration

Latent heat of evaporation of water is

2.51 ~~2,250~~ J g^{-1} - (2.4 J g^{-1} °C^{-1})

2.45 (~~~2.5~~ kJ g^{-1}) at 25°C = $2.45 \frac{MJ}{L}$

STEP 6. Determine heat transfer by conduction + convection

$$HT = 2 * K * \frac{\Delta T}{BL}$$

Where:

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2 = Each leaf has two sides and both sides can transfer heat.

K = The thermal conductivity coefficient of air, which is 0.026 W m^{-1} C^{-1}. This varies slightly with temperature and moisture content of the air, but we will consider it a constant.

dT = Delta temperature - difference between leaf temperature and air temperature. This can be either positive or negative (unit = C or K).

BL = Air boundary layer. This is the thickness of the stagnant air layer next to the leaf (unit = meters). Note that typical boundary layers are 0.0001 to 0.01 meters. BL thickness is determined by:

BL = $0.004\sqrt{\text{leaf width / air velocity (meters per second)}}$

Typical values for air velocity (wind speed) are:

0.1 m s-1 = still air

10.0 m s-1 = high wind (22 mph)

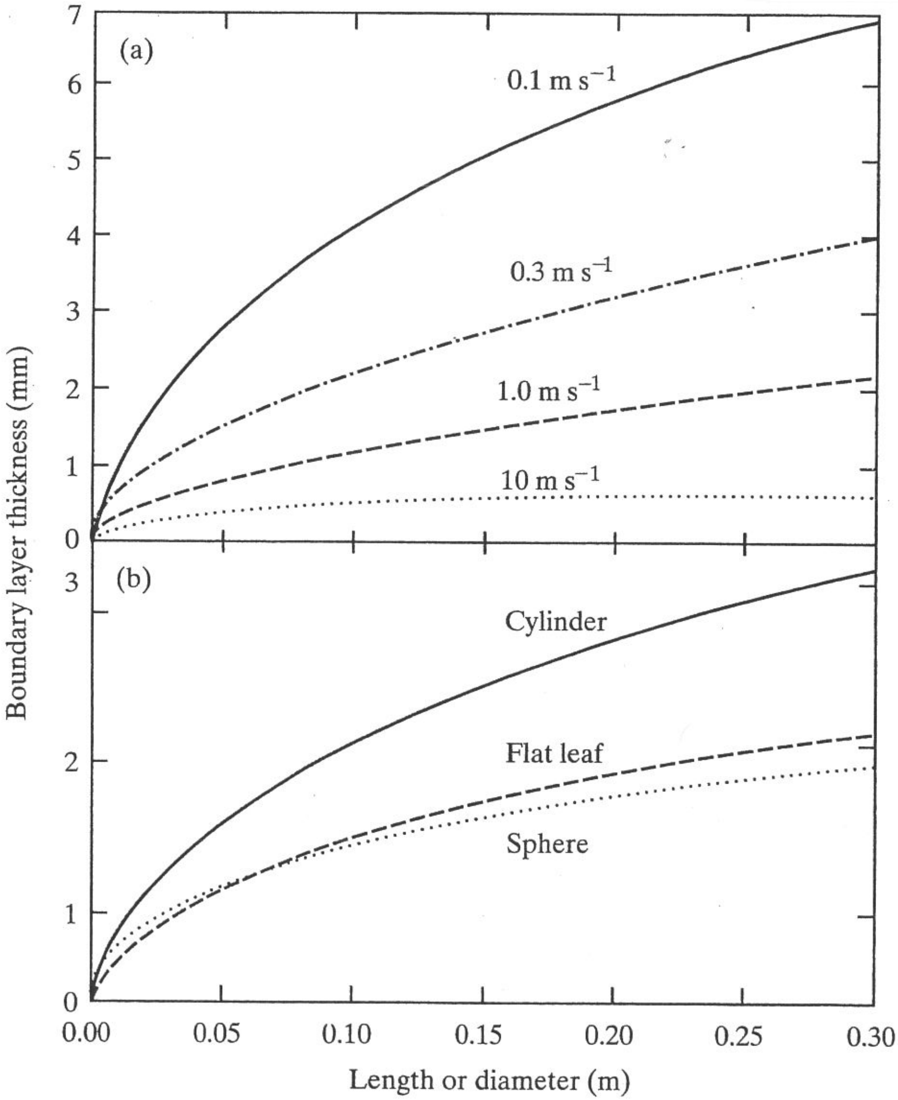


Figure 7-7. Mean thickness of the air boundary layer (a) adjacent to a flat leaf at various wind speeds indicated next to the curves and (b) adjacent to objects of three different shapes at a wind speed of 1 m s^{-1}. The length for a flat leaf represents the mean distance across it in the direction of the wind; the diameter is used for the bluff bodies represented by cylinders and spheres. Values were determined using Equations 7.10 through 7.12. Note that 1.0 m s^{-1} equals 3.6 km hour^{-1} or 2.2 mile hour^{-1}.

for example, increasing the spacing between nursery stock leads to greater wind exposure for each tree and hence to sturdier trees with larger trunk diameters. Interestingly, mechanically shaking closely spaced young trees in a nursery for about 20 seconds per day simulates wind effects and can increase stem diameter, avoiding the spacing requirements between plants necessary for movement to occur naturally by frictional interactions with the wind.

Buttresses at the base of tree trunks and roots are more common on the windward (upwind) side. Such a location more effectively resists upsetting forces caused by wind than if the buttresses or enhanced root growth were on the leeward (downwind) side, because the tensile strength of wood is greater than is its compressional strength. Another consequence of a prevailing wind direction is "flag trees," where branches occur mainly toward the leeward side. Most of these effects of wind on stem morphology are hormonally mediated. At the extreme of sporadic high winds, such as occur in gales (wind speeds of 17–21 m s^{-1}, corresponding to 61–76 km $hour^{-1}$ or 38–47 miles $hour^{-1}$), form drag can cause stems to be permanently displaced from their upright position. This process is termed "lodging" for various cereal crops. To prevent lodging, breeding programs have developed rice, wheat, barley, and oat genotypes with shorter, sturdier stems. Wind is also one of the major factors in the ecology of forests, forming gaps in the canopy by uprooting trees, creating special microhabitats by distributing leaf litter, and influencing reproductive success by dispersing pollen, spores, and certain seeds.

7.2B. Air Boundary Layers

Wind speed affects the thickness of the air boundary layer next to a leaf or other aerial plant part. Because boundary layers influence heat exchange and hence the temperature of the shoot, any process in a shoot depending on temperature can be affected by the wind speed. Also, every molecule entering or leaving a leaf in the gas phase must cross the air boundary layer next to its surface.

A boundary layer is a region of a fluid next to a solid that is dominated by the shearing stresses originating at the surface of the solid; such layers arise for any solid in a fluid, such as a leaf in air. Adjacent to the leaf is a laminar sublayer of air (Fig. 7-6), where air movement is predominantly parallel to the leaf surface. Air movement is arrested at the leaf surface and has increasing speed at increasing distances from the surface. Diffusion

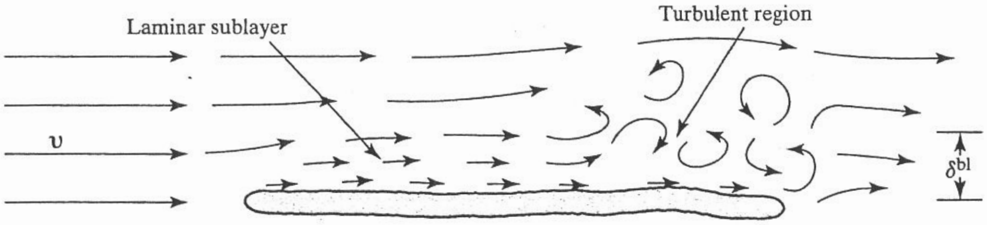
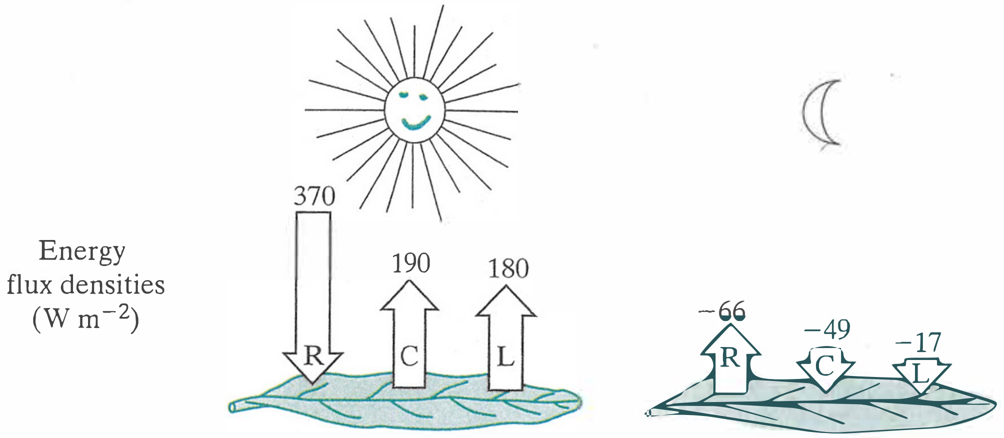


Figure 7-6. Schematic illustration of originally nonturbulent air (straight arrows in upwind side on left) flowing over the top of a flat leaf, indicating the laminar sublayer (shorter straight arrows), the turbulent region (curved arrows), and the effective boundary layer thickness, δ^{bl}. The length of an arrow indicates the relative speed, and the curvature indicates the local direction of air movement. A similar airflow pattern occurs on the lower leaf surface.



Leaf temperature	25°C	−1°C
Air temperature	20°C	1°C
Relative humidity	50%	92%
Wind speed	0.8 m s^{-1}	0.4 m s^{-1}
Water flux density	4 mmol m^{-2} s^{-1} transpired	0.2 mm frost in 9 hours

(a) (b)

Figure 7-10. Energy budget for an exposed leaf (a) at midday and (b) at night with frost. The flux densities in W m^{-2} are indicated for net radiation (R); conduction/convection (C), also referred to as sensible heat; and latent heat (L). By convention net radiation gain by a leaf is considered positive, as are heat conduction/convection and latent heat losses (Eqs. 7.1 and 7.2).

7.3B. Heat Flux Density for Dew or Frost Formation

So far we have considered the usual case where c_{wv}^{e} is greater than c_{wv}^{ta}, which results in a net loss of water from a leaf and a consequent dissipation of heat. However, when the turbulent air is warmer than the leaf and also has a high relative humidity, the water vapor concentration in the turbulent air can be greater than that in the leaf (in Chapter 2, Section 2.4C we noted that the water vapor concentration and partial pressure at saturation increase rapidly with temperature, e.g., Fig. 2-16; also, see values for P_{wv}^* and c_{wv}^* in Appendix I). If c_{wv}^{ta} is greater than the water vapor concentration in a leaf, then a net diffusion of water vapor occurs toward the leaf. This can increase c_{wv} at the leaf surface, and it may reach c_{wv}^*, the saturation value, which is most likely at night. If c_{wv}^{ta} is greater than this c_{wv}^*, dew—or frost, if the leaf temperature is below freezing—can form as water vapor diffuses toward the leaf and then condenses onto its surface, which is cooler than the turbulent air. Condensation resulting from water emanating from the soil is sometimes called "distillation," with the term "dew" being reserved for water coming from the air above.

Condensation of water vapor leads to heat gain by a leaf. In particular, water condensation is the reverse of the energy-dissipating process of water evaporation, so the heat gain per unit amount of water condensed is the heat of vaporization of water at the temperature of the leaf, H_{vap}. Because the condensation is on the leaf surface, the diffusion is across the air boundary layers of thickness δ^{bl} that are present on each side of a leaf. To describe the rate of heat gain per unit area accompanying the water vapor condensation

ENERGY REQUIRED TO CHANGE WATER FROM ICE $\xrightarrow{\text{to}}$ LIQUID $\xrightarrow{\text{to}}$ GAS

	CHANGE OF STATE	CHANGE IN TEMPERATURE	ENERGY REQUIRED MJ per Kg
SOLID (ICE) 0°C ⇄ LIQUID (WATER) 0°C	YES	NO	0.34
LIQUID 0°C ⇄ LIQUID 100°C	NO	YES	0.42
LIQUID 100°C ⇄ GAS 100°C	YES	NO	2.25

THE LATENT HEAT OF EVAPORATION

LIQUID → GAS

25°C 25°C

$2.45 \frac{MJ}{Kg}$

←

(WHEN WATER CONDENSES THIS IS CALLED THE LATENT HEAT OF CONDENSATION)

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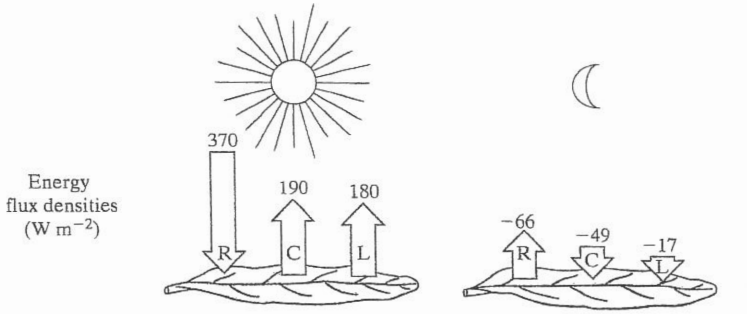

Leaf temperature	25°C	−1°C
Air temperature	20°C	1°C
Relative humidity	50%	92%
Wind speed	$0.8\ m\ s^{-1}$	$0.4\ m\ s^{-1}$
Water flux density	$4\ mmol\ m^{-2}\ s^{-1}$ transpired	0.2 mm frost in 9 hours
	(a)	(b)

Figure 7-10. Energy budget for an exposed leaf (a) at midday and (b) at night with frost. The flux densities in $W\ m^{-2}$ are indicated for net radiation (R); conduction/convection (C), also referred to as sensible heat; and latent heat (L). By convention net radiation gain by a leaf is considered positive, as are heat conduction/convection and latent heat losses (Eqs. 7.1 and 7.2).

Table 7-1. Representative Values for the Various Terms in the Net Radiation Balance of an Exposed Leaf[a]

Condition	Global irradiation, S ($W\ m^{-2}$)	Absorbed solar irradiation, $a(1+r)S$ ($W\ m^{-2}$)	Temperature of surroundings (°C)	Sky temperature (°C)	Absorbed infrared, $a_{IR}\sigma[(T^{surr})^4 + (T^{sky})^4]$ ($W\ m^{-2}$)	Leaf temperature (°C)	Emitted infrared, $2e_{IR}\sigma\ (T^{leaf})^4$ ($W\ m^{-2}$)	Net radiation ($W\ m^{-2}$)
1. Sea level on cloudless day	840	605	20	−20	624	25	859	370
2000 m on cloudless day	920	662	10	−25	555	24	847	370
2. Silvery leaf ($a = 0.50$) at 2000 m on cloudless day	920	552	10	−25	555	13	737	370
3. Sea level on cloudy night	0	0	1	1	614	1	614	0
4. Sea level on cloudless night	0	0	1	−20	530	−9 *	530	0
	0	0	1	−20	530	−1	596	−66

[a] Equation 7.8 is used, taking a as 0.60 (except where indicated), r as 0.20, and both a_{IR} and e_{IR} as 0.96 (see text for interpretations).

a = FRACTION ABSORBED SHORTWAVE

r = ALBEDO OF SOIL SURFACE

$a_{IR} = e_{IR}$ = ABSORPTIVITY = EMISSIVITY

* −9 = NO WIND

−1 = WITH WIND